

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

Cambridge Ordinary Level

## **MARK SCHEME for the October/November 2014 series**

### **4037 ADDITIONAL MATHEMATICS**

**4037/13**

Paper 1, maximum raw mark 80

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1	$a = 3$ $b = 2$ $c = 4$	<b>B1</b> <b>B1</b> <b>B1</b>	
2	$x^2 = 16$ or $y^2 - 4y + 3 = 0$  $x = \pm 4$ $y = 1, 3$ Points $(-4, 1)$ and $(4, 3)$ Line $AB = \sqrt{8^2 + 2^2}$ $= \sqrt{68}$ or $2\sqrt{17}$	<b>M1</b>  <b>A1</b> <b>A1</b>  <b>M1</b> <b>A1</b>	for correct elimination of one variable and attempt to form a quadratic equation in $x$ or $y$ .  for use of Pythagoras theorem allow either form
3	(i) $n(A) = 2$ $n(B) = 3$ $n(C) = 0$  (ii) $A \cup B = \{-1, -2, -3, 3\}$  (iii) $A \cap B = \{-2\}$  (iv) $\xi$ , 'the universal set', $\mathbb{R}$ , 'real numbers', $\{x : x \in \}$	<b>B1</b> <b>B1</b> <b>B1</b>  <b>B1</b>  <b>B1</b>  <b>B1</b>	<b>B0</b> for $n(2)$ , $\{2\}$ , $\{0\}$ , $\emptyset$ , $\{\}$ etc.
4	(a) $\tan x = -\frac{5}{3}$  $x = 121.0^\circ, 301.0^\circ$  (b) $\sin\left(3y + \frac{\pi}{4}\right) = \frac{1}{2}$  $3y + \frac{\pi}{4} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$  $3y = -\frac{\pi}{12}, \frac{7\pi}{12}, \frac{23\pi}{12}, \frac{31\pi}{12}$  $y = \frac{7\pi}{36}, \frac{23\pi}{36}, \frac{31\pi}{36}$ (0.611, 2.01 and 2.71)	<b>M1</b>  <b>A1</b> <b>A1ft</b>  <b>M1</b>  <b>A1</b>  <b>DM1</b>  <b>A1, A1</b>	Correct statement or $\tan x = -1.67$  <b>A1</b> for either correct solution <b>ft</b> from <i>their</i> first solution  for dealing correctly with cosec and attempt to solve subsequent equation  for $\frac{\pi}{6}, \frac{5\pi}{6}, \text{ or } \frac{13\pi}{6}, \text{ or } \frac{17\pi}{6}$  for correct order of operations  <b>A1</b> for one correct solution <b>A1</b> for both the other correct solutions and no others in range.

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<p>5 (a) (i)</p> $\begin{pmatrix} 12 & 2 & 1 \\ 9 & 3 & 0 \\ 8 & 5 & 1 \\ 11 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.4 \\ 0.45 \end{pmatrix} = \begin{pmatrix} 7.25 \\ 5.70 \\ 6.45 \\ 6.30 \end{pmatrix}$ <p>or <math>(0.5 \ 0.4 \ 0.45) \begin{pmatrix} 12 &amp; 9 &amp; 8 &amp; 11 \\ 2 &amp; 3 &amp; 5 &amp; 2 \\ 1 &amp; 0 &amp; 1 &amp; 0 \end{pmatrix}</math></p> <p><math>= (7.25 \ 5.70 \ 6.45 \ 6.30)</math></p> <p>(ii) 25.70</p>		<p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A2,1,0</b></p> <p><b>B1</b></p>	<p>for correct compatible matrices in the correct order. Allow 1 error in each matrix. Allow if done in cents</p> <p>for a correct method for multiplying their matrices to obtain an appropriate 4 by 1 or 1 by 4 matrix.</p> <p><b>A2</b> all correct or <b>A1</b> 3 correct elements.</p> <p>Allow 25.7</p>
<p>(b)</p> <p><math>\mathbf{Y} = \mathbf{X}^{-1}</math> or <math>\mathbf{Y} = \mathbf{X}^{-1}\mathbf{I}</math></p> $\mathbf{Y} = \frac{1}{22} \begin{pmatrix} 1 & -4 \\ 5 & 2 \end{pmatrix} \text{ or } \begin{pmatrix} \frac{1}{22} & -\frac{4}{22} \\ \frac{5}{22} & \frac{2}{22} \end{pmatrix}$ <p>Alternative method:</p> $\begin{pmatrix} 2 & 4 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p><math>2a + 4c = 1, 2b + 4d = 0</math> <math>-5a + c = 0, -5b + d = 1</math></p> <p>leading to <math>= \frac{1}{22} \begin{pmatrix} 1 &amp; -4 \\ 5 &amp; 2 \end{pmatrix}</math> oe</p>		<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>for matrix algebra</p> <p>for <math>\frac{1}{22} \begin{pmatrix} &amp; \\ &amp; \end{pmatrix}</math></p> <p>for <math>k \begin{pmatrix} 1 &amp; -4 \\ 5 &amp; 2 \end{pmatrix}</math></p> <p>for a complete method using simultaneous equations</p> <p><math>a = \frac{1}{22}</math> and <math>c = \frac{5}{22}</math> or <math>b = -\frac{4}{22}</math> and <math>d = \frac{2}{22}</math></p> <p>for correct matrix</p>

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<p><b>6 (i)</b></p> $\cos 0.9 = \frac{6}{OC} \text{ or } \frac{OC}{\sin 0.9} = \frac{12}{\sin(\pi - 1.8)}$ $OC = \frac{6}{\cos 0.9} = 9.652\dots$ <p>or <math>OC = \frac{12 \sin 0.9}{\sin(\pi - 1.8)} = 9.652\dots</math></p> <p><b>(ii)</b></p> $\text{Perimeter} = (0.9 \times 12) + 9.652 + (12 - 9.652)$ $= 22.8$ <p><b>(iii)</b></p> $\text{Area} = \left( \frac{1}{2} \times 12^2 \times 0.9 \right) - \left( \frac{1}{2} \times 9.652^2 \sin(\pi - 1.8) \right)$ $64.8 - 45.36$ $= 19.4 \text{ to } 19.5$ <p>Alternative Method:</p> $\frac{1}{2}(12 - 9.652) \times 9.652 \times \sin 1.8$ $\frac{1}{2}12^2(0.9 - \sin 0.9)$ $11.04 + 8.40$ $\text{Area} = 19.4 \text{ to } 19.5$		<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>for correct use of cosine, sine rule, cosine rule or any other valid method</p> <p>for manipulating correctly to <math>OC = 9.652(35\dots)</math> Must have 4<sup>th</sup> figure (or more) for rounding</p> <p>for arc length for attempt to add the correct lengths</p> <p>for area of sector, allow unsimplified for area of isosceles triangle <math>\frac{1}{2}(9.65(2\dots))^2 \sin(\pi - 1.8)</math> or <math>\frac{1}{2}(12 \times 6 \tan 0.9)</math> or <math>\frac{1}{2}(12 \times 9.65(2\dots) \times \sin 0.9)</math>, allow unsimplified. for answer in range 19.4 to 19.5</p> <p>for area of triangle <math>ACB</math>, unsimplified for area of segment, unsimplified answer in range 19.4 to 19.5</p>
<p><b>7</b></p> $1 + 2 \log_5 x = \log_5(18x - 9)$ $\log_5 5 + \log_5 x^2 = \log_5(18x - 9)$ $5x^2 = 18x - 9$ $(5x - 3)(x - 3) = 0$ $x = \frac{3}{5}, 3$		<p><b>B1, B1</b></p> <p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p>	<p><b>B1</b> for dealing with '1', <b>B1</b> for dealing with '2'</p> <p>for a correct use of addition or subtraction of logarithms</p> <p>for elimination of logarithms to form a 3 term quadratic and for solution of quadratic for both <math>x</math> values</p>

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<p><b>8 (i)</b></p> $f'(x) = \left(x \times \frac{3x^2}{x^3}\right) + (\ln x^3)$ $= 3 + 3 \ln x, = 3(1 + \ln x)$ <p>or <math>f(x) = 3x \ln x</math></p> $f'(x) = \left(3x \times \frac{1}{x}\right) + 3 \ln x,$ $= 3(1 + \ln x)$		<p><b>M1</b> <b>B1</b> <b>A1</b> <b>B1</b> <b>M1</b> <b>A1</b></p>	<p>for differentiation of a product for differentiation of <math>\ln x^3</math> for simplification to gain <u>given answer</u> for use of <math>\ln x^3 = 3 \ln x</math> for differentiation of a product for simplification to gain <u>given answer</u></p>
<p><b>(ii)</b></p> $\int 3(1 + \ln x) dx = x \ln x^3 \text{ or } 3x \ln x$ $\int 1 + \ln x dx = \frac{1}{3} x \ln x^3 \text{ or } x \ln x$		<p><b>M1</b> <b>A1</b></p>	<p>for realising that differentiation is the reverse of integration and using <b>(i)</b></p>
<p><b>(iii)</b></p> $x \ln x - \int 1 dx \text{ or } \left[\frac{1}{3} x \ln x^3\right] - \int 1 dx$ $[x \ln x - x]_1^2 \text{ or } \left[\frac{1}{3} x \ln x^3 - x\right]_1^2$ $= 2 \ln 2 - 2 + 1$ $= -1 + \ln 4$		<p><b>DM1</b> <b>DM1</b> <b>A1</b></p>	<p>for using answer to <b>(ii)</b> and subtracting <math>\int 1 dx</math> dependent on M mark in <b>(ii)</b> for correct application of limits from correct working</p>
<p><b>9 (a)</b></p> $5^p = 625, \text{ so } p = 4$ ${}^4C_1 5^{p-1}(-q) = -1500$ $4 \times 125(-q) = -1500$ $q = 3$ ${}^4C_2 5^{p-2} q^2 = r$ $r = 1350$ <p><b>(b)</b></p> ${}^{12}C_3 (2x)^9 \left(\frac{1}{4x^3}\right)^3$ <p>Term is 1760</p>		<p><b>B1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>DM1</b> <b>A1</b></p>	<p><i>their p</i> substituted in <math>{}^pC_1 5^{p-1}(-q)</math> or in <math>{}^pC_1 5^{p-1}(-qx)</math> unsimplified  <i>their p</i> and <i>q</i> substituted in <math>{}^pC_2 5^{p-2}(-q)^2</math> or <math>{}^pC_2 5^{p-2}(-qx)^2</math> unsimplified  for identifying correct term for attempt to evaluate correct expression must be evaluated</p>

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<p><b>10 (a)</b></p>	$\frac{5^x}{5^{2(3y-2)}} = 1 \text{ or } \frac{3^x}{3^{3(y-1)}} = 3^4 \text{ oe}$ $x = 6y - 4$ $x = 3y + 1$ <p>Leads to <math>x = 6, y = \frac{5}{3}</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>for obtaining one correct equation in powers of 5, 3, 25, 27 or 81</p> <p>for <math>x = 6y - 4</math> oe linear equation</p> <p>for <math>x = 3y + 1</math> oe linear equation</p> <p>for attempt to solve linear simultaneous equations which have been obtained correctly for both.</p>
<p><b>(b)</b></p>	<p>Using the cosine rule:</p> $(1 + 2\sqrt{3})^2 = (2 + \sqrt{3})^2 + 2^2 - 4(2 + \sqrt{3})\cos A$ $\cos A = \frac{(13 + 4\sqrt{3}) - (7 + 4\sqrt{3}) - 4}{-4(2 + \sqrt{3})} \text{ oe}$ $\cos A = \frac{-1}{2(2 + \sqrt{3})} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$ $\cos A = -1 + \frac{\sqrt{3}}{2}$	<p><b>M1</b></p> <p><b>DM1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p>	<p>for correct substitution in cosine rule, may use in form of <math>\cos A = \dots</math></p> <p>for attempt to make <math>\cos A</math> subject and simplify</p> <p>for rationalisation.</p>

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<p><b>11 (i)</b></p> $\frac{dy}{dx} = (x+5)2(x-1) + (x-1)^2$ $\frac{dy}{dx} = (x-1)(3x+9)$ <p>When <math>\frac{dy}{dx} = 0</math></p> $x = 1$ $x = -3$ <p>Alternative method:</p> $y = x^3 + 3x^2 - 9x + 5$ $\frac{dy}{dx} = 3x^2 + 6x - 9$ <p>When <math>\frac{dy}{dx} = 0</math></p> $x = 1$ $x = -3$		<p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>for differentiation of a product, allow unsimplified correct</p> <p>for equating to zero and solution of quadratic</p> <p>for expansion of brackets and differentiation of each term of a 4 term cubic</p> <p>for equating to zero and solution of 3 term quadratic</p> <p>from correct quadratic equation</p> <p>from correct quadratic equation</p>
<p><b>(ii)</b></p> $\int x^3 + 3x^2 - 9x + 5 dx$ $= \frac{x^4}{4} + x^3 - \frac{9x^2}{2} + 5x (+c)$		<p><b>M1</b></p> <p><b>A2,1,0</b></p>	<p>for correct attempt to obtain and integrate a 4 term cubic</p> <p><b>A2</b> for 4 correct terms or <b>A1</b> for 3 correct terms</p>
<p><b>(iii)</b></p> $\left[ \frac{x^4}{4} + x^3 - \frac{9x^2}{2} + 5x \right]_{-5}^1$ $= \left( \frac{1}{4} + 1 - \frac{9}{2} + 5 \right) - \left( \frac{625}{4} - 125 - \frac{225}{2} - 25 \right)$ $= 108$		<p><b>M1</b></p> <p><b>A1</b></p>	<p>for correct substitution of limits 1 and -5 for <i>their</i> (ii)</p>
<p><b>(iv)</b></p> <p>When <math>x = -3, y = 32</math></p> <p><math>k &gt; 32</math></p>		<p><b>M1</b></p> <p><b>A1</b></p>	<p>for realising that the <math>y</math>-coordinate of the maximum point is needed.</p>